

**PROGRAMMING MATHCAD TO SOLVE TWO SIMULTANEOUS 2ND ORDER
DIFFERENTIAL EQUATIONS FOR PROJECTILE MOTION
WITH AIR RESISTANCE**

This handout helps you to organize the two 2nd order differential equations governing the motion of a projectile into a format suitable for solution using Mathcad. All the symbols and terminology are defined in the other project handouts, and in class.

GOVERNING EQUATIONS OF MOTION

Horizontal motion: $-F_D \cos(\mathbf{b}) = ma_x = m \frac{d^2x}{dt^2}$

Vertical motion: $-F_D \sin(\mathbf{b}) - mg = ma_y = m \frac{d^2y}{dt^2}$

with: $F_D = C_D \frac{1}{2} \mathbf{r} v^2 A$

$C_D = 0.5$ = coefficient of drag (experimentally determined)

\mathbf{r} = air density (units?)

A = cross-sectional area of the projectile (balloon)

$$v = \text{speed of balloon} = v = \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}$$

$$\cos(\mathbf{b}) = \frac{v_x}{v} = \frac{\left(\frac{dx}{dt} \right)}{\sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}}$$

$$\sin(\mathbf{b}) = \frac{v_y}{v} = \frac{\left(\frac{dy}{dt} \right)}{\sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}}$$

FOR MATHCAD we need the equations in the form:

$$\frac{d^2x}{dt^2} = fn\left(const, \frac{dx}{dt}, \frac{dy}{dt}\right)$$

$$\frac{d^2y}{dt^2} = fn\left(const, \frac{dx}{dt}, \frac{dy}{dt}\right)$$

SUBSTITUTE and REARRANGE to get:

$$\frac{d^2x}{dt^2} = \frac{-C_D r A}{2m} \left\{ \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right\}^{1/2} \frac{dx}{dt}$$

and

$$\frac{d^2y}{dt^2} = -g - \frac{C_D r A}{2m} \left\{ \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right\}^{1/2} \frac{dy}{dt}$$

FOR MATHCAD let us set the following:

$$C_o = \frac{C_D r A}{2m}$$

$$p_1 = \frac{dx}{dt}$$

$$p_2 = \frac{d^2x}{dt^2}$$

$$p_3 = \frac{dy}{dt}$$

$$p_4 = \frac{d^2y}{dt^2}$$

hence:

$$D(t, p) = \begin{bmatrix} dx/dt \\ d^2x/dt^2 \\ dy/dt \\ d^2y/dt^2 \end{bmatrix} = \begin{bmatrix} p_1 \\ -C_o(\sqrt{p_1^2 + p_3^2})p_1 \\ p_3 \\ -g - C_o(\sqrt{p_1^2 + p_3^2})p_3 \end{bmatrix}$$

with initial conditions:

$$\begin{bmatrix} x_i \\ dx_i/dt \\ y_i \\ dy_i/dt \end{bmatrix} = \begin{bmatrix} 0 \\ v_i \cos(\theta_i) \\ 0 \\ v_i \sin(\theta_i) \end{bmatrix}$$

rkfixed output is in tabular form with columns:

index	t	x	dx/dt	y	dy/dt
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